Matter wave solitons at finite temperatures

B. Jackson, C. F. Barenghi, and N. P. Proukakis

School of Mathematics and Statistics, University of Newcastle upon Tyne, NE1 7RU, United Kingdom

We consider the dynamics of a dark soliton in an elongated harmonically trapped Bose-Einstein condensate. A central question concerns the behavior at finite temperatures, where dissipation arises due to the presence of a thermal cloud. We study this problem using coupled Gross-Pitaevskii and N-body simulations, which include the mean field coupling between the condensate and thermal cloud. We find that the soliton decays relatively quickly even at very low temperatures, with the decay rate increasing with rising temperature.

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1. INTRODUCTION

Dark solitons represent a ubiquitous feature of nonlinear systems[1], and their experimental realization in atomic Bose-Einstein condensates (BECs)[2,3,4,5] has opened up new possibilities for their study. The stability of the soliton, and its dependence upon the geometry of the condensate, is a problem of considerable interest. In three dimensions a soliton is dynamically unstable to undulations in its transverse profile. This so-called "snake" instability leads to the soliton decaying to vortex lines or rings, and has been observed experimentally[4,5]. This instability, however, can be suppressed by confining the condensate much more strongly in the transverse direction than longitudinally, so that the condensate is highly elongated along the axial direction.

A second decay mechanism arises from the fact that at non-zero temperatures the condensate coexists with a noncondensed cloud composed of thermally excited quasiparticles. Interactions of the soliton with the thermal cloud leads to dissipation, so that the soliton loses energy and accelerates. Experiments[2] suggest that the timescale for this decay can be relatively short, on the order of 10 ms, so has important consequences for the dynamics. This decay process has previously been studied theoretically by Fedichev

et al.[6] and Muryshev *et al.*[7] by considering the reflection of thermal excitations from the soliton. The role of quantum fluctuations was addressed in Dziarmaga *et al.*[8].

In this paper we describe the use of numerical simulations[9] to model a soliton at finite temperatures in a highly elongated condensate, where the trap frequency in the radial direction, $\omega_\perp$, greatly exceeds that along the axial direction, ω_z. At $T = 0$ the soliton is predicted[10,11], on the basis of the Gross-Pitaevskii (GP) equation, to oscillate longitudinally at a frequency of $\omega_z/\sqrt{2}$, which involves continuous undamped soliton-sound interactions[13]. We follow the motion of the soliton as a function of time, finding the expected oscillation along the axial direction. At finite temperature the energy loss of the soliton due to the presence of the thermal cloud leads to an increase in the amplitude of the oscillation. We investigate this soliton decay process as a function of temperature, and find that it is appreciable even at very low T.

2. THEORY

Our treatment of finite temperatures is based on the ZNG formalism[12], where we solve the following coupled equations

$$i\hbar\frac{\partial \Psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m} + V + gn_c + 2g\tilde{n}\right)\Psi, \tag{1}$$

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{m}\cdot\boldsymbol{\nabla} f - \boldsymbol{\nabla} U \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f = 0. \tag{2}$$

The former is a generalized GP equation for the condensate wavefunction $\Psi(\boldsymbol{r}, t)$, and the latter is a Boltzmann equation for the thermal cloud phase space density $f(\boldsymbol{p}, \boldsymbol{r}, t)$. Here the condensate and thermal cloud densities are defined as $n_c = |\Psi|^2$ and $\tilde{n} = \int f\, d\boldsymbol{p}/h^3$ respectively, while $g = 4\pi\hbar^2 a/m$ parametrizes the mean field interactions between atoms of mass m with scattering length a. The effective potential is $U = V + 2g(n_c + \tilde{n})$, with $V = m(\omega_\perp^2 r^2 + \omega_z^2 z^2)/2$ representing the harmonic trap confining the atoms. Note that for the purposes of this paper we neglect collisions between the atoms, and the coupling between the condensate and thermal cloud is purely mean field in nature. For the parameters considered here, inclusion of collisions is expected to slightly increase the damping rate, but this will be considered explicitly in future work.

The initial condition for our simulations is found by self-consistently solving (1) in imaginary time to find the ground state wavefunction, which is coupled to a thermal cloud described by a Bose distribution $f(\boldsymbol{p}, \boldsymbol{r}) =$

$\{\exp[(p^2/2m + U - \mu)/k_B T] - 1\}^{-1}$, where μ is the condensate chemical potential. The methods for solving Eqs. (1) and (2) for the subsequent finite temperature dynamics are described by Jackson and Zaremba[9]. A difference with respect to this earlier work is that here the GP equation (1) is solved using cylindrical coordinates with a Crank-Nicholson time-stepping scheme. Since the system is cylindrically symmetric the problem effectively reduces to two dimensions, improving the efficiency of the simulations.

3. RESULTS

Finite temperature simulations have been performed for $N = 2 \times 10^4$ ^{87}Rb atoms in a trap with frequencies $\omega_\perp = 2\pi \times 2500\,\mathrm{Hz}$ and $\omega_z = 2\pi \times 10\,\mathrm{Hz}$, resulting in a cloud that is highly elongated along the axial direction. The critical temperature for Bose condensation is close to that for an ideal gas in the thermodynamic limit[14], $T_c^0 = 0.941\hbar(\omega_\perp^2 \omega_z N)^{1/3}/k_B$, which for these parameters corresponds to $T_c^0 = 486\,\mathrm{nK}$. At time $t = 0$ the equilibrium configuration for a given temperature is imprinted with a dark soliton, by multiplying the condensate wavefunction $\Psi(\boldsymbol{r})$ with[13] $\Psi_s(z) = \beta \tanh(\beta z/\xi) + iv/c$, where $\beta = \sqrt{1-(v/c)^2}$, $\xi = \hbar/\sqrt{mgn}$ is the condensate healing length, and $c = \sqrt{gn/(2m)}$ is the speed of sound for density n. For the benefit of the following discussion, it is useful to note that for a condensate of uniform density n_0 the soliton energy is given by[1] $E_s = 4\hbar c n_0 \beta^3/3$, while the density at the soliton minimum (equal to the difference between the background density and the depth of the soliton) is[14] $n_s = n_0 v^2/c^2$. So an increase in v corresponds to a decrease in the soliton energy and depth, which both tend to zero as $v \to c$.

For $T = 0$ the dynamics are simply given by the GP equation (1) with $\tilde{n} = 0$. The black lines in Fig. 1 plot the subsequent evolution of the density at early times, where the soliton oscillates along the axial direction. The position of the soliton minimum, z_s, is plotted in Fig. 2 (a), illustrating that the soliton oscillates at constant amplitude with a frequency of $\omega_z/\sqrt{2}$, as expected[10,11,13]. The parameter n_s, representing the density at the soliton minimum as labelled in Fig. 1 (a), acquires its maximum value at the center of the trap and is zero at the turning points of the soliton motion. Fig. 2 (b) plots n_s, divided by the central density of the condensate without a soliton, n_0. The peak value of this parameter (attained at the trap centre) remains constant, as expected for undamped motion.

The corresponding density cross-sections for $T = 150\,\mathrm{nK}$ are plotted with gray lines in Fig. 1, where one sees that, even during the first oscillation, the presence of the thermal cloud leads to an increase in the amplitude of

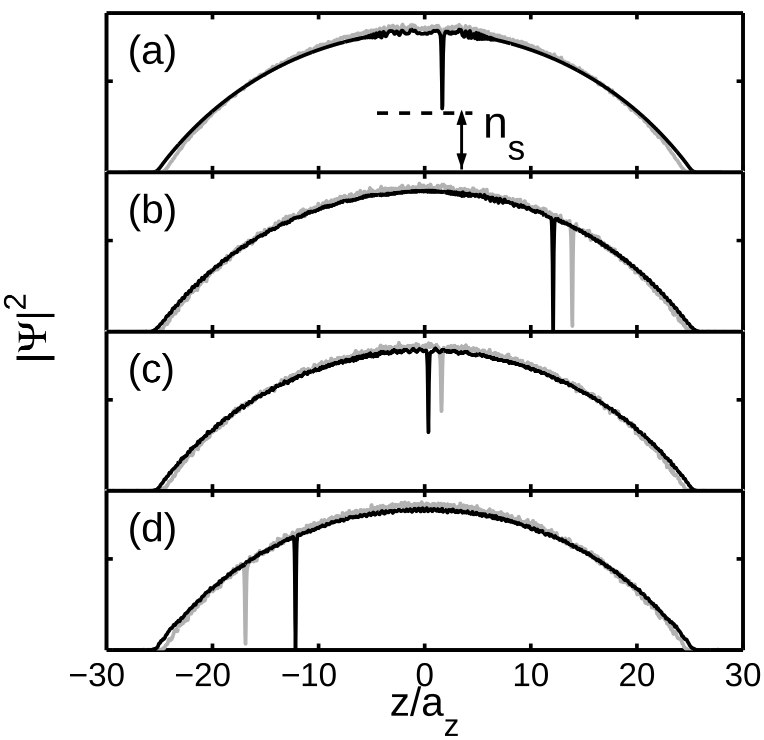

Fig. 1. Density $|\Psi|^2$ along an axial cross-section of the condensate, showing the evolution of a dark soliton with initial velocity $v = 0.5c$. The black lines plot the densities for $T = 0$, while the gray lines are for $T = 150\,\text{nK}$, at times of (a) $\omega_z t = 0.2$, (b) $\omega_z t = 2.2$, (c) $\omega_z t = 4.4$, and (d) $\omega_z t = 6.7$. The axial position z is in units of $a_z = \sqrt{\hbar/(m\omega_z)}$, while the density is in arbitrary units. The density at the soliton minimum, n_s, is labelled in (a).

the oscillation (Fig. 1 (b) and (d)), in addition to a significant decrease in the depth of the soliton when it is at the trap center (Fig. 1 (c)). The steady rise in oscillation amplitude is also shown in Fig. 2 (a), while the decrease in soliton depth at $z_s = 0$ (corresponding to an increase in n_s) is illustrated in Fig. 2 (b). This accompanies an increase in velocity at $z_s = 0$, and reflects an energy decay as the soliton interacts with the thermal cloud.

Simulations have also been performed at other temperatures, and one finds less damping at lower T (as illustrated for $T = 100\,\text{nK}$ in Fig. 2) and more damping at higher T, as expected. However, it is remarkable that appreciable damping occurs even for these relatively low temperatures ($T < 0.3T_c$) , which contrasts with the situation for collective modes where the effects of finite temperature tend to be significant only at higher

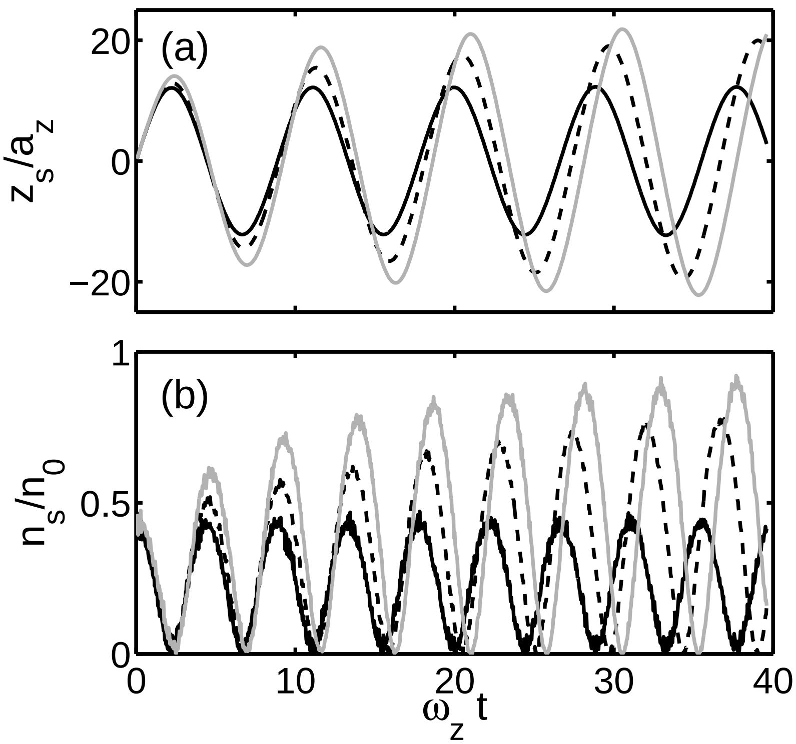

Fig. 2. (a) Axial position (in units of a_z) of the soliton minimum as a function of time, where initially $v = 0.5c$. The curves show results for different temperatures, with $T = 0$ (solid black), $T = 100\,\mathrm{nK}$ (dashed), and $T = 150\,\mathrm{nK}$ (gray). (b) Density at the soliton minimum n_s (divided by the central density of the condensate without a soliton, n_0) as a function of time, where the curves are labelled as in (a).

temperatures[15,16,17]. Another noteworthy feature relates to the time dependence of the amplitude, the increase of which appears to saturate as it approaches the radius of the condensate. This effect has also been observed for $v = 0.25c$, where the initial oscillation amplitude is smaller, and hence the saturation does not occur until later times.

4. CONCLUSIONS

We have performed numerical simulations to study the dynamics of dark solitons in a Bose-Einstein condensate at different temperatures. Our numerics for zero temperature reproduce soliton oscillations along the axial direction at constant amplitude and at a frequency of $\omega_z/\sqrt{2}$. However, at

finite temperature, mean field coupling between the condensate and thermal cloud is shown to lead to a lower frequency oscillation with a steadily increasing amplitude. This corresponds to the soliton losing energy to the thermal cloud, with a rate that increases with rising initial temperature.

Note that this energy transfer cannot be considered as a heating of the thermal cloud, since temperature is not defined during the simulation due to a lack of rethermalizing collisions. Future calculations will study the influence of these collisional processes, as well as modelling the soliton decay for the experimental parameters of Burger *et al.*[2]. A further motivation for future work is to ascertain the feasibility of Proukakis *et al.*[18], where a parametric driving scheme was proposed for stabilization of a soliton against a phenomenologically modelled thermal cloud. We plan to revisit this problem with our more detailed finite temperature simulations.

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